# PORTFOLIO MANAGEMENT CLASS 3

# CLASS WORK COVERAGE

To streamline our learning process, I've categorized the questions we'll tackle in class into four distinct groups:

1. ***Classic:*** *These questions are exactly as presented in your book, providing a familiar foundation.*
2. ***Transformed:*** *Here, we've converted book questions into multiple-choice format to enhance your analytical skills.*
3. ***Adapted:*** *These are similar to book questions but with altered numbers or names, presented as multiple-choice questions for varied practice.*
4. ***Original:*** *These are entirely new questions not found in your book, designed to challenge and expand your understanding.*

*This structure will help us navigate through a range of problems, ensuring a comprehensive grasp of the material. Looking forward to our next session!*

| Q. No. | Type | Book | Page No. | Q No. |
|---|---|---|---|---|
| Case Study 2 | ***Transformed*** | CW Q BOOK | 45 | 4 |
| Case Study 3 | ***Transformed*** | CW Q BOOK | 45 | 5 |
| Case Study 4 | ***Transformed*** | CW Q BOOK | 46 | 6 |
| Case Study 5 | ***Transformed*** | CW Q BOOK | 46 | 7 |
| Case Study 6 | ***Transformed*** | CW Q BOOK | 46 | 8 |

# CASE STUDY 2

| Years | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
|---|---|---|---|---|---|---|---|---|---|---|
| Security 1 (Return %) | 15 | 10 | 12 | 8 | 18 | 16 | 20 | 24 | 16 | 14 |
| Security 2 (Return %) | 24 | 20 | 18 | 14 | 22 | 26 | 12 | 28 | 16 | 15 |

**Question 1:**

Based on the historical rates of return for Security 1 and Security 2, calculate the **Covariance** of the two securities.

**A.** 10.30

**B.** 8.95

**C.** 7.85

**D.** 9.45

**Question 2:**

What is the **Correlation Coefficient** between Security 1 and Security 2?

**A.** 0.42

**B.** 0.41

**C.** 0.50

**D.** 0.45

**Answer:**

**Question 1:**

D is correct.

**Question 2:**

B is correct.

**Explanation:**

**Calculation of Covariance**

| Year | $R_1$ | Deviation $(R_1-\overline{R_1})$ | Deviation $(R_1-\overline{R_1})^2$ | $R_2$ | Deviation $(R_2-\overline{R_2})$ | Deviation $(R_2-\overline{R_2})^2$ | Product of deviations |
|---|---|---|---|---|---|---|---|
| 1 | 15 | -0.3 | 0.09 | 24 | 4.5 | 20.25 | -1.35 |
| 2 | 10 | -5.3 | 28.09 | 20 | 0.5 | 0.25 | -2.65 |
| 3 | 12 | -3.3 | 10.89 | 18 | -1.5 | 2.25 | 4.95 |
| 4 | 8 | -7.3 | 53.29 | 14 | -5.5 | 30.25 | 40.15 |
| 5 | 18 | 2.7 | 7.29 | 22 | 2.5 | 6.25 | 6.75 |
| 6 | 16 | 0.7 | 0.49 | 26 | 6.5 | 42.25 | 4.55 |
| 7 | 20 | 4.7 | 22.09 | 12 | -7.5 | 56.25 | -35.25 |
| 8 | 24 | 8.7 | 75.69 | 28 | 8.5 | 72.25 | 73.95 |
| 9 | 16 | 0.7 | 0.49 | 16 | -3.5 | 12.25 | -2.45 |
| 10 | 14 | -1.3 | 1.69 | 15 | -4.5 | 20.25 | 5.85 |
| | 153 | | Σ= 200.10 | 195 | | Σ=262.50 | 94.50 |

$$\overline{R_1} = \frac{153}{10} = 15.30$$

$$\overline{R_2} = \frac{195}{10} = 19.50$$

$$\text{Covariance} = \frac{\sum_{i=1}^{N} [R_1 - \overline{R_1}][R_2 - \overline{R_2}]}{N} = 94.50/10 = 9.45$$

Standard Deviation of Security 1

$$\sigma_1 = \sqrt{\frac{(R_1 - \overline{R_1})^2}{N}}$$

$$\sigma_1 = \sqrt{\frac{200.10}{10}} = \sqrt{20.01}$$

$$\sigma_1 = 4.47$$

Standard Deviation of Security 2

$$\sigma_2 = \sqrt{\frac{(R_2 - \overline{R_2})^2}{N}}$$

$$\sigma_2 = \sqrt{\frac{262.50}{10}} = \sqrt{26.25}$$

$$\sigma_2 = 5.12$$

*Alternatively, Standard Deviation of securities can also be calculated as follows:*

| Year | $R_1$ | $R_1^2$ | $R_2$ | $R_2^2$ |
|---|---|---|---|---|
| 1 | 15 | 225 | 24 | 576 |
| 2 | 10 | 100 | 20 | 400 |
| 3 | 12 | 144 | 18 | 324 |
| 4 | 8 | 64 | 14 | 196 |
| 5 | 18 | 324 | 22 | 484 |
| 6 | 16 | 256 | 26 | 676 |
| 7 | 20 | 400 | 12 | 144 |
| 8 | 24 | 576 | 28 | 784 |
| 9 | 16 | 256 | 16 | 256 |
| 10 | 14 | 196 | 15 | 225 |
| | 153 | 2541 | 195.00 | 4065 |

Standard deviation of security 1:

$$\sigma_1 = \sqrt{\frac{N\sum R_1^2 - (\sum R_1)^2}{N^2}}$$

$$\sigma_1 = \sqrt{\frac{10 \times 2541 - (153)^2}{10^2}} = \sqrt{\frac{25410 - 23409}{100}}$$

$$\sigma_1 = \sqrt{\frac{2001}{100}} = \sqrt{20.01}$$

$$\sigma_1 = 4.47$$

Standard deviation of security 2:

$$\sigma_2 = \sqrt{\frac{N\sum R_2^2 - (\sum R_2)^2}{N^2}}$$

$$\sigma_2 = \sqrt{\frac{10 \times 4065 - (195)^2}{10^2}} = \sqrt{\frac{40650 - 38025}{100}}$$

$$\sigma_2 = \sqrt{\frac{2625}{100}} = \sqrt{26.25}$$

$$\sigma_2 = 5.12$$

Correlation Coefficient

$$r_{12} = \frac{Cov}{\sigma_1 \sigma_2} = \frac{9.45}{4.47 \times 5.12} = 0.413$$

# CASE STUDY 3

Mayuri is interested to construct a Portfolio of Securities X and Y. She has collected the following information:

| | X | Y |
|---|---|---|
| Expected Return (ER) | 19% | 23% |
| Risk ( σ ) | 14% | 18% |

Mayuri has 5 Portfolio options of X and Y as follows:

**i.** 50% of funds in each X and Y

**ii.** 75% of funds in X and 25% in Y

**iii.** 25% of funds in X and 75% in Y

**iv.** 60% of funds in X and 40% in Y

**v.** 35% of funds in X and 65% in Y

Suppose if Co-efficient of correlation (r) between X and Y is 0.16, you are required to calculate the following:

**Question 1:**

**Expected Return** for Portfolio Option (i), where 50% of funds are in X and 50% in Y?

**A.** 19.00%

**B.** 21.00%

**C.** 20.00%

**D.** 22.00%

**Question 2:**

**Risk Factor (σ)** associated with Portfolio Option (iv), where 60% of funds are in X and 40% in Y?

**A.** 12.25%

**B.** 12.07%

**C.** 11.91%

**D.** 14.48%

**Question 3:**

Which portfolio is best from the **point of view of Risk**?

**A.** Portfolio (v)

**B.** Portfolio (iv)

**C.** Portfolio (ii)

**D.** Portfolio (i)

**Question 4:**

Which portfolio is best from the **point of view of Return**?

**A.** Portfolio (iii)

**B.** Portfolio (ii)

**C.** Portfolio (i)

**D.** Portfolio (v)

**Answer:**

**Question 1:**

B is correct.

**Question 2:**

C is correct.

**Question 3:**

B is correct.

**Question 4:**

A is correct.

**Explanation:**

We have $E_p = W_1E_1 + W_3E_3 + \ldots\ldots\ldots W_nE_n$

and for standard deviation $\sigma^2_p = \sum_{i=1}^{n}\sum_{j=1}^{n} w_i w_j$

$$\sigma^2_p = \sum_{i=1}^{n}\sum_{j=1}^{n} w_i w_j \rho_{ij}\, \sigma_i\, \sigma_j$$

Two asset portfolio

$\sigma^2_p = w^2_1\sigma^2_1 + w^2_2\sigma^2_2 + 2\, w_1w_2\sigma_1\sigma_2\rho_{12}$

Or

$\sigma_p = \sqrt{w_1\sigma_1 + w_2\sigma_2 + 2\, w_1w_2\sigma_1\sigma_2\rho_{12}}$

**Substituting the respective values we get,**

i. **50% of funds in each of X and Y**

Ep = 0.50 X 19% + 0.50 X 23%= 21%

$\sigma^2_p = (0.50)^2(14\%)^2 + (0.50)^2(18\%)^2 + 2(0.50)(0.50)(0.16)(14\%)(18\%)$

$\sigma^2_p = 49 + 81 + 20.16 = 150.16$

$\sigma_p$= 12.25%

ii. **75% in X and 25% in Y**

Ep = 0.75 X 19% + 0.25 X 23% = 20%

$\sigma^2_p = (0.75)^2(14\%)^2 + (0.25)^2(18\%)^2 + 2(0.75)(0.25)(0.16)(14\%)(18\%)$

$\sigma^2_p = 110.25 + 20.25 + 15.12 = 145.62$

$\sigma_p$= 12.07%

**iii. 25% in X and 75% in Y**

Ep = 0.25 X 19% + 0.75 X 23% =22%

$\sigma^2_p = (0.25)^2(14\%)^2 + (0.75)^2(18\%)^2 + 2(0.25)(0.75)(0.16)(14\%)(18\%)$

$\sigma^2_p = 12.25 + 182.25 + 15.12 = 209.62$

$\sigma_p = 14.48\%$

**iv. 60% in X and 40% in Y**

Ep = 0.60 X 19% + 0.40 X 23% = 20.60%

$\sigma^2_P = (0.60)^2(14\%)^2 + (0.40)^2(18\%)^2 + 2(0.60)(0.40)(0.16)(14\%)(18\%)$

$\sigma^2_P = = 70.56 + 51.84 + 19.35 = 141.75$

$\sigma_p = 11.91\%$

**v. 35% in X and 65% in Y**

Ep = 0.35 X 19% + 0.65 X 23% = 21.60%

$\sigma^2_P = (0.35)^2(14\%)^2 + (0.65)^2(18\%)^2 + 2(0.35)(0.65)(0.16)(14\%)(18\%)$

$\sigma^2_P = 24.01 + 136.89 + 18.35 = 179.25$

$\sigma_p = 13.39\%$

| Portfolio | (i) | (ii) | (iii) | (iv) | (v) |
|---|---|---|---|---|---|
| Return | 21.00 | 20.00 | 22.00 | 20.60 | 21.60 |
| σ | 12.25 | 12.07 | 14.48 | 11.91 | 13.39 |

In the terms of return, we see that portfolio (iii) is the best portfolio.

In terms of risk we see that portfolio (iv) is the best portfolio.

# CASE STUDY 4

Consider the following information on two stocks, A and B:

| Year | Return on A (%) | Return on B (%) |
|---|---|---|
| 2016 | 10 | 12 |
| 2017 | 16 | 18 |

**Question 1:**

What is the **Expected Return** for the portfolio containing stocks A and B in the proportion of 40% and 60%, respectively?

**A.** 12.5%

**B.** 13.5%

**C.** 14.2%

**D.** 15%

**Question 2:**

What is the **Standard Deviation** of return for Stock A?

**A.** 2%

**B.** 3%

**C.** 4%

**D.** 5%

**Question 3:**

What is the **Covariance** of returns between stocks A and B?

**A.** 10

**B.** 9

**C.** 8

**D.** 7

**Question 4:**

What is the **Correlation Coefficient** between the returns of stocks A and B?

**A.** 1

**B.** 0.5

**C.** 0.75

**D.** 0.25

**Question 5:**

What is the **Risk** of a portfolio containing stocks A and B in the proportion of 40% and 60%, respectively?

**A.** 2%

**B.** 4%

**C.** 5%

**D.** 3%

**Answer:**

**Question 1:**

C is correct.

**Question 2:**

B is correct.

**Question 3:**

B is correct.

**Question 4:**

A is correct.

**Question 5:**

D is correct.

**Explanation:**

**i. Expected return of the portfolio A and B**

E (A) = (10 + 16) / 2 = 13%

E (B) = (12 + 18) / 2 = 15%

$$Rp = \sum_{i-I}^{N} X_i R_i = 0.4(13) + 0.6(15) = 14.2\%$$

**ii. Stock A:**

Variance = $0.5\ (10 - 13)^2 + 0.5\ (16 - 13)^2 = 9$

Standard deviation = $\sqrt{9} = 3\%$

**Stock B:**

Variance = $0.5\ (12 - 15)^2 + 0.5\ (18 - 15)^2 = 9$

Standard deviation = 3%

### iii. Covariance of stocks A and B

$Cov_{AB} = 0.5\ (10 - 13)\ (12 - 15) + 0.5\ (16 - 13)\ (18 - 15) = 9$

### iv. Correlation of coefficient

$$r_{AB} = \frac{CoV_{AB}}{\sigma_A \sigma_B} = \frac{9}{3 \times 3} = 1$$

### v. Portfolio Risk

$$\sigma_P = \sqrt{X^2{}_A \sigma^2{}_A + X^2{}_B \sigma^2{}_B + 2X_A X_B \left(\sigma_A \sigma_B \sigma_{AB}\right)}$$

$$= \sqrt{(0.4)^2 (3)^2 + (0.6)^2 (3)^2 + 2(0.4)(0.6)(3)(3)(1)}$$

$$= \sqrt{1.44 + 3.24 + 4.32} = 3\%$$

# CASE STUDY 5

Following information is available on Return (%) of shares of two companies A and B:

| Probabilities | Return of A | Return of B |
|---|---|---|
| 0.05 | 6 | 8 |
| 0.20 | 12 | 18 |
| 0.50 | 20 | 28 |
| 0.20 | 24 | 34 |
| 0.05 | 30 | 44 |

**Question 1:**

What is the **Expected Return** for Share A, based on the given probabilities and returns?

**A.** 5.46%

**B.** 19%

**C.** 15%

**D.** 13%

**Question 2:**

What is the **Standard Deviation** for Share A?

**A.** 3.50%

**B.** 7.68%

**C.** 5.46%

**D.** 6.12%

**Question 3:**

What is the **Expected Return** for Share B?

**A.** 19%

**B.** 22%

**C.** 25%

**D.** 27%

**Question 4:**

What is the **Standard Deviation** for Share B?

**A.** 7.68%

**B.** 6.12%

**C.** 5.46%

**D.** 4.80%

**Question 5:**

What is the **Risk of the Portfolio** at a 70:30 ratio of Share A and Share B?

**A.** 5.46%

**B.** 6.12%

**C.** 7.68%

**D.** 6.12%

**Answer:**

**Question 1:**

B is correct.

**Question 2:**

C is correct.

**Question 3:**

D is correct.

**Question 4:**

A is correct.

**Question 5:**

B is correct.

**Explanation:**

**i. For Share A**

| *Prob p* | *Return of A (RA)* | *R = RA x p* | *Deviation DA = RA -R* | *DA²* | *DA² x p* |
|---|---|---|---|---|---|
| 0.05 | 6 | 0.3 | -13 | 169 | 8.45 |
| 0.20 | 12 | 2.4 | -7 | 49 | 9.80 |
| 0.50 | 20 | 10.0 | 1 | 1 | 0.50 |
| 0.20 | 24 | 4.8 | 5 | 25 | 5.00 |
| 0.05 | 30 | 1.5 | 11 | 121 | 6.05 |
| Total | | 19.0 | | | 29.80 |

Std Deviation = $\sqrt{29.80}$ = 5.46 %

For A : Expected Return = 19%, Std Deviation = 5.46 %

**For Share B :**

| Prob p | Return of B (RB) | R = RB x p | Deviation DB = RB -R | DB $^2$ | DB$^2$ x p |
|---|---|---|---|---|---|
| 0.05 | 8 | 0.4 | -19 | 361 | 18.05 |
| 0.20 | 18 | 3.6 | -9 | 81 | 16.20 |
| 0.50 | 28 | 14.0 | 1 | 1 | 0.50 |
| 0.20 | 34 | 6.8 | 7 | 49 | 9.80 |
| 0.05 | 44 | 2.2 | 17 | 289 | 14.45 |
| Total | | 27.0 | | | 59.00 |

Std Deviation = $\sqrt{59.00}$ = 7.68 %

For B : Expected Return = 27%, Std Deviation = 7.68%

| *Prob p* | *DA* | *DB* | *DA x DB* | *PX DA x DB* |
|---|---|---|---|---|
| 0.05 | -13 | -19 | 247 | 12.35 |
| 0.20 | -7 | -9 | 63 | 12.60 |
| 0.50 | 1 | 1 | 1 | 0.50 |
| 0.20 | 5 | 7 | 35 | 7.00 |
| 0.05 | 11 | 17 | 187 | 9.35 |
| Total | | | | $Cov_{AB}$=41.80 |

$Corr_{AB}$ = $Cov_{AB}$ / ( $\sigma_A$ X $\sigma_B$) = 41.80 / ( 5.46 x 7.68) = 41.80 /41.9328 = 0.9968

**ii. Risk of the portfolio at 70:30 ratio =**

$$= \sigma_P = \sqrt{X^2_A\sigma^2_A + X^2_B\sigma^2_B + 2X_AX_B(\sigma_A\sigma_B\sigma_{AB})}$$

= [(0.7)² (5.46) ² + (0.3) ² (7.68) ² + 2 x (5.46)x(7.68) (0.9968)] $^5$

= [29.80 x 0.49 + 59 x 0.09 +2 x 8.778= 37.468$^5$ = 6.12

# CASE STUDY 6

Suppose Mr. X in a world where there are only two assets, gold and stocks. He is interested in investing his money in one, the other or both assets. Consequently he collects the following data on the returns on the two assets over the last six years.

| | Gold | Stock Market |
|---|---|---|
| Average return | 8% | 20% |
| Standard deviation | 25% | 22% |
| Correlation | - | 0.4 |

**Question 1:**

Mr. X is constrained to pick just one asset. Which one would he choose based on the provided data?

A. Gold

B. Stocks

C. Both Gold and Stocks

D. Neither Gold nor Stocks

**Question 2:**

Mr. Y, a friend of Mr. X, argues that Mr. X is ignoring the big payoffs that he can get on gold. How would Mr. X alleviate his concern?

**A.** By choosing gold with a higher proportion

**B.** By ignoring gold entirely

**C.** By diversifying into both assets

**D.** By concentrating all funds in stocks

**Question 3:**

How would a portfolio composed of equal proportions in gold and stocks perform in terms of mean and variance?

**A.** The mean return will be 14%, and the variance will be 387.25

**B.** The mean return will be 14%, and the variance will be 12.93%

**C.** The mean return will be 12%, and the variance will be 12.93%

**D.** The mean return will be 10%, and the variance will be 131.75

**Question 4:**

What effect will the supply of gold, being negatively correlated with the stock market, have on Mr. X's portfolio?

**A.** It will not affect the portfolio

**B.** It will increase the risk of the portfolio

**C.** It will make gold less desirable

**D.** It will make gold more desirable

**Answer:**

**Question 1:**

B is correct.

**Question 2:**

C is correct.

**Question 3:**

A is correct.

**Question 4:**

C is correct.

**Explanation:**

i. Mr. X would pick the stock market portfolio, since it dominates gold on both average return and standard deviation.

ii. The higher possible returns on gold are balanced by the lower possible returns at other times. Note that the average return on gold is much less than that on the stock market.

iii. The expected return on this portfolio would be (8+20)/2 = 14%. The variance would equal $(0.5)^2(25)^2 + (0.5)^2(22)^2 - 2(0.5)(0.5)(25)(22)(0.4) = 387.25$; the standard deviation equals 19.68%

iv. If the supply of gold is negatively correlated with the level of the market, and the price of gold is inversely related to the supply of gold, there is a positive correlation between the return on the market and the return on gold. This would make gold less desirable, since it does not help as much in reducing portfolio variance. The optimal amount to invest in gold would drop.